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LETTER TO THE EDITOR

Superconductor glass phase in strongly disordered bulk systems

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Abstract. The superconducting behaviour of highly disordered bulk electronic systems close to the metal–insulator transition (MIT) is considered. It is argued that these systems are frustrated even in the absence of a magnetic field. If disorder fluctuations are larger than some critical value a superconductor glass phase is predicted. Technically the results are based on Thouless–Anderson–Palmer-like mean-field equations that are derived to describe the proposed glass transition.

During the last 10 years glassy phases in granular superconductors [1, 2] and high-T_c superconductors (vortex glass [3]) have been proposed. In both of these systems the existence of a magnetic field played an essential role for the formation of the glass. Here we show that under special circumstances disorder fluctuation in disordered superconductors can cause a glass phase even in the absence of a magnetic field. (For related work see [4, 5].)

It is currently believed that a necessary condition for the formation of glasses is the presence of frustration in the system under consideration. Frustration was introduced as a concept to clarify the physics behind the spin glass transition [6]. Its main effect is the prevention of a conventional macroscopic ordered state. In spin glasses, e.g., ferromagnetic and antiferromagnetic order is destroyed. In general frustration is not sufficient to cause a glassy state. Without disorder the transition temperature is typically zero [7, 8] or the formation of a frustrated but periodic macroscopic state is observed dependent on the model [7, 9]. In nature, frustration and disorder are not independent entities. In simple spin glass models [10] the change from a ferromagnetic transition to a spin glass transition is caused by increasing the width of the distribution of coupling parameters, i.e., the disorder. Here we argue that in the problem of superconductivity in strongly disordered interacting systems a similar situation arises if the metal to insulator transition (MIT) is approached.

In this letter we first give a heuristic argument for why a glassy phase is expected in general. Then a more technical derivation is presented. We begin by noting that non-magnetic disorder is able to suppress superconductivity in interacting electronic systems [11]. The basic idea is that disorder decreases the diffusivity of the electrons. Consequently two electrons spend more time around each other, which in turn effectively increases the Coulomb pseudopotential between the electrons. The net result is that the BCS pairing potential, Γ, is effectively decreased. This effect is largest at long length scales because the diffusion process is a long-wavelength phenomenon. This implies that with increasing disorder, Γ^{eff} becomes too small to sustain macroscopic superconductivity even though locally a superconducting state is preferred, i.e., the system is frustrated on long length

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scales. The connection to the conventional concept of frustration can be made clearer if we consider the effective-field theory describing the T_c suppression mechanism [12]. An effective partition function for a given realization of the disorder can be derived, completely analogous to [12, 11]. It is given by

$$Z = \int D(\Delta^*, \Delta) \exp -\beta \Big(\Gamma^{-1} \int \mathrm{d}x\, \Delta^*(x)\Delta(x) + \tfrac{1}{2} h(T) \int \mathrm{d}x\, (\Delta^*(x)\Delta(x))^2 - \int \mathrm{d}x \int \mathrm{d}x'\, \Delta^*(x) \sum_n \hat{C}_n(x, x')\Delta(x') \Big). \tag{1}$$

The spin singlet BCS interaction term Γ is treated explicitly. The effects of Coulomb interaction and disorder are contained in $\hat{C}_n(x, x') = \langle \Psi(x, n)\Psi^*(x', -n)\Psi(x, -n)\Psi^*(x', n) \rangle$ and in principle in $h(T)$. The thermodynamic average in $C(x, x') = \sum_n \hat{C}_n(x, x')$ is over a reference system without a bare Cooper interaction and $\Psi(x, n)$ is the Grassmann variable at spatial point x and Matsubara frequency $\omega_n = (2n + 1)\pi T$. The summation over frequencies is cut off at the Debye frequency ω_D. In deriving (1) the complex auxiliary field $\Delta(x)$ was restricted to the lowest Matsubara frequency $\Delta(x) = \Delta(x, n = \pi T)$, which is appropriate for a qualitative description of glasses in quantum systems [13]. In this paper we focus on fluctuations in $C(x, x')$ only and therefore $h(T)$ is taken to be the disorder averaged quantity. Since we are interested in very low-temperature phenomena we use [14, 15] $h(T) = 2\pi(N_F/\pi^2)7/8\zeta(3)\Gamma^{-4} f(T)$ ($f(T) \propto 1/T^2$ for $T \to \infty$, $f(T) = \mathrm{O}(1)$ for $T \to 0$), with N_F the density of states at the Fermi level.

In the disorder-averaged theory the saddlepoint of Z gives the Landau free energy for superconductivity. $\Gamma^{-1}\Delta(x) = \Delta^{\mathrm{Gap}}$ is the BCS order parameter for homogeneous superconductivity. Treating the disorder renormalization carefully (for a review see [11]) the disorder average $\int \overline{C}(x)\,\mathrm{d}x = \overline{C}(q = 0)$ can be calculated. Schematically, $\overline{C}(q = 0) = \log(\omega_D/T)/(2N_F + \delta\Gamma \log(\omega_D/T)) \equiv J_0$, where $\delta\Gamma$ is the renormalization to the BCS coupling Γ due to disorder and electron–electron interaction. The condition for the transition temperature is therefore given by $\Gamma^{-1} - J_0 = 0$.

For the qualitative physical discussion let us assume for the moment that also for the non-averaged Z the essential physics is represented by the ordinary saddle point. Later we show that this is not quite correct. The implicit equation for the transition temperature T_c is then $\Delta(x) - \Gamma^{-1} \int \mathrm{d}^3x' (\overline{C}(|x - x'|) + \delta C(x, x'))\Delta(x') = 0$. Here $C(x, x')$ is split in an obvious way into disorder-averaged and fluctuation parts. It is a Hermitian matrix and can be written as $\delta C(x, x') = |\delta C(x, x')| \exp \mathrm{i}\Phi(x, x')$. The phase $\Phi(x, x')$ and the amplitude are random quantities due to the disorder. Note that the quenched random phase $\Phi(x, x')$ is not a function of $|x - x'|$ alone but depends on x, x' separately. For the critical behaviour it is essential that the random phase cannot be written as $\Phi_1(x) - \Phi_2(x')$, which would lead to a uniformly frustrated X–Y spin glass by performing a gauge transformation as in the Mattis spin glass model [16]. The uniformly frustrated X–Y spin glass was suggested for inhomogeneously disordered superconductors in [5], which also discusses the possibility of glassy behaviour in the absence of a magnetic field. In contrast to [5] in our case the interplay of disorder and the electron interaction causes a quenched random phase [17]. An eventual glass transition belongs therefore to the gauge glass universality class [18, 19].

The condition for the transition temperature is $\det(\Gamma^{-1}\delta(x - x') - C(x, x')) = 0$. It is sufficient that the largest eigenvalue of $C(x, x')$ is equal to Γ^{-1}. In general the eigenvalue cannot be calculated. To proceed, we take advantage of δC being a random quantity with respect to different realizations of the disorder. Therefore it is reasonable to approximate $\delta C(x, x')$ as a random matrix in the mathematical sense [22]. This approximation will enable us to arrive at a quantitative estimate of the effect of disorder fluctuations close

to the MIT. The largest eigenvalue of $\overline{C}(|x-x'|)$ is J_0 (corresponding to the eigenvector $\Delta(x)=\Delta$, independent of x). Since we are only interested whether the disorder effects are able to dominate the average behaviour it is sufficient to concentrate only on this eigenvalue, neglecting all other (smaller) eigenvalues. In this case the eigenvalue spectrum of $C(x,x')$ consists of the Wigner semicircle with radius $2J=(2/\sqrt{n})\sqrt{\int dx'\,\overline{\delta C(x,x')\delta C(x',x)}}$ and an isolated eigenvalue at $J(J_0/J+J/J_0)$ if $J_0>J$. If $J<J_0$ the spectrum is given by the Wigner semicircle alone [20]. The result for J was derived in Fourier space for finite volume and then expressed in spatial integrals. n is the electron density.

The two cases $J_0>J$ and $J_0<J$ describe two qualitatively different regimes. For $J_0>J$ the isolated eigenvalue is the largest one. The disordered system transition temperature T_c^d satisfies $\Gamma J_0\approx 1-(\Gamma J)^2$ (we will see later that this is not quite the correct equation to order J^2). The resulting phase can be considered as a superconductor with small spatial variations of the order parameter, if $J\ll J_0$ [21]. With increasing disorder $\delta\Gamma$ and J are increasing, J_0 is decreasing. If the increase in $\delta\Gamma$ is stronger than the increase in J, at some disorder the condition $\delta\Gamma/\Gamma=1/(1-(\Gamma J)^2)$ is fulfilled. This leads to $T_c^d=0$. The superconductivity is completely suppressed. Experimentally these are the cases, where the suppression of T_c is not too close to the MIT and therefore ΓJ usually can be neglected.

A qualitative new situation develops if $J_0\leqslant J$. In this case the eigenvalue spectrum of $C(x,x')$ is symmetric and also the distribution of the eigenvectors is symmetric under very general mathematical conditions [22]. The real and imaginary parts of $\Delta(x')$ are therefore positive or negative with equal probability for different realizations of the disorder, i.e. $\overline{\langle\Delta\rangle}(x)=0$, but there is a non-vanishing Edwards–Anderson order parameter $\overline{\langle\Delta^*(x)\rangle\langle\Delta(x)\rangle}$. This means physically the resulting phase is a superconductor glass with zero energy gap. Since we know that disorder fluctuations become large close to the MIT (for a review see [23]), we expect J to increase when the MIT is approached. Therefore if the new phase exists it is most likely to be found near the MIT. On general grounds it cannot be decided whether there is a direct transition from a disordered superconductor to the superconductor glass by increasing the disorder, or whether there is a gap between the superconductor phase and the emergence of the glass.

To discuss these ideas on a more quantitative level, we derive a mean-field theory, which is at least correct to order J^2. We show that in principle J can become large enough to create a glassy phase. The mean-field theory given here is analogous to the Thouless–Anderson–Palmer [24] mean-field theory for spin glasses. Starting from (1) it is easy to derive TAP-like equations using a method proposed by Plefka [25]. Two source fields $Y^*(x)$ and $Y(x)$ which couple to $\Delta(x)$ and $\Delta^*(x)$ respectively are introduced and a Legendre transformation is performed $\beta W=-\log Z+\int dx\langle\Delta^*\rangle(x)Y(x)+\int dx\,Y^*(x)\langle\Delta\rangle(x)$. $\langle\ldots\rangle$ indicates a thermodynamic average for a given realization of the disorder. W is expanded around the local energy contribution: $\Gamma^{-1}\int dx\,\Delta^*(x)\Delta(x)+\frac{1}{2}Th(T)\int dx\,(\Delta^*(x)\Delta(x))^2$ up to second order in $C(x,x')$. After using a saddle-point approximation for the local reference system with source fields the free energy functional becomes [17]

$$\begin{aligned}\beta W=\beta\int dx\,[\Gamma^{-1}\langle\Delta^*\rangle(x)\langle\Delta\rangle(x)+\tfrac{1}{2}\beta h(T)(\langle\Delta^*\rangle(x)\langle\Delta\rangle(x))^2]\\ -\int dx\int dx'[\beta\langle\Delta^*\rangle(x)C(x,x')\langle\Delta\rangle(x')\\ -\beta^2\delta C(x,x')\delta C(x',x)\langle\langle\Delta^*(x)\Delta(x)\rangle\rangle\langle\langle\Delta^*(x')\Delta(x')\rangle\rangle].\end{aligned}\tag{2}$$

$\langle\langle\ldots\rangle\rangle$ is the second thermodynamic cumulant calculated in the local reference system. Contributions from thermodynamic fluctuations (leading to a shift in the superconducting

transition temperature), local temperature fluctuations ($\delta C(x, x)$) and terms which do not contribute to the equation for the transition temperature (higher order in $\langle\Delta\rangle$) are neglected in (2). They do not change the qualitative conclusions. The condition for linear stability is given by $\delta W/\delta(\langle\Delta\rangle) = \delta W/\delta(\langle\Delta^*\rangle) = 0$:

$$\left(\Gamma^{-1} + Th(T)\Gamma^3 \int \mathrm{d}x'\, \delta C(x, x')\delta C(x', x)\right)\langle\Delta\rangle(x) - \int \mathrm{d}x'\, C(x, x')\langle\Delta\rangle(x') = 0$$

and a similar equation for $\langle\Delta^*\rangle(x)$. In strongly disordered superconductors the typical scale for the variation of $\langle\Delta\rangle(x)$ is the mean free path l. $h(T)$ can therefore be written $h(T) = (\hat{T} + 8\pi/3\hat{D})/(\hat{T} + 8\pi\hat{D})^3$ ($\hat{D} = D\tau/l^2$, $\hat{T} = T\tau$, τ is the elastic scattering time) using the approximation $D\partial^2/\partial x_i^2\langle\Delta\rangle(x) \approx D/l^2\langle\Delta\rangle(x)$. The eigenvalue equation,

$$\Gamma^{-1} - J_0 - J^2/J_0 + Th(T)\Gamma^3 nJ^2 = 0 \qquad \text{for } J < J_0$$
$$\Gamma^{-1} - 2J + Th(T)\Gamma^3 nJ^2 = 0 \qquad \text{for } J > J_0$$

can now be solved if J is known. A simple qualitative reasoning reveals the structure of the necessary perturbation theory. For non-interacting electrons, C can be related to the diffusion process in an inhomogeneous system. Since $J^2 \sim \overline{(\delta C)^2}$, it is schematically given by $(\delta[\int \mathrm{d}^3q \sum_n 1/(2\omega_n + Dq^2)])^2$. The most singular part at the MIT is related to the fluctuation of the diffusion constant (or conductivity) in the denominator. In the perturbation theory terms up to $(\delta Dq_1^2)(\delta Dq_2^2)$ must be included. The most singular part is given by [17]

$$\frac{4\tau^4}{N_F^4 n}\int_{q_1,q_2,q_3} T^2 \sum_{\omega_1,\omega_2} \frac{(D(q_1^2+q_2^2)\tau + \omega_+\tau)(D(q_2^2+q_3^2)\tau + \omega_+\tau)}{(Dq_1^2\tau + \omega_+\tau)^2(Dq_3^2\tau + \omega_+\tau)^2(Dq_2^2\tau + 2\omega_1\tau)(Dq_2^2\tau + 2\omega_2\tau)}.$$

The diffusons with frequency ω_1, ω_2 respectively represent the contributions of the conductivity fluctuation at different frequencies [26]. It is interesting to note that the largest contribution can formally be written as $J^2 \propto T^2 \sum_{\omega_1,\omega_2} \overline{\delta N(r_1, \omega_+)^2}\ \overline{\delta N(r_2, \omega_+)^2}$, where $\omega_+ = \omega_1 + \omega_2$. The maximum eigenvalue is related to the product of the local density of state fluctuations at the different points. The divergent contribution to J^2 close to the MIT is, after integrating over momenta, $\propto (\tau^4/N_F^4)T^2 \sum_{\omega_1,\omega_2}(D_+\tau)^{-3}(\omega_+\tau)^{-1}$. We have introduced $D_+ = D_0F(\omega_+)$ as the diffusivity at Matsubara frequency ω_+ in the critical regime. D_0 normalizes the critical spectrum D_+. An explicit expression for $F(\omega_+)$ in interacting systems is not known. For non-interacting systems an approximate $F(\omega_+)$ can be determined in the region near the MIT [27]. It has the scaling form $F(\omega_+, \epsilon) = \epsilon D^*(\omega_+/\epsilon^3)$, with $D^*(x) = x^{1/3}$ for $x \to 0$ and $D^*(x) =$ constant for $x \to \infty$. ϵ is proportional to the difference of the impurity density ρ from the critical density ρ_c. In the explicit calculation the frequency integrals cannot be done analytically. At $\epsilon = 0$, J^2 has a logarithmic divergence for $T \to 0$. At $T = 0$, J^2 diverges $\propto 1/\epsilon^{3/2}$ for $\epsilon \to 0$.

Figure 1 shows schematically the two generic phase diagrams. The strength of the disorder is given by some parameter μ. Macroscopic superconductivity is suppressed at μ_c. The glass phase appears at μ_g. For $\mu_g > \mu_c$ the glass and the macroscopic superconductor are separated as shown in the upper part of figure 1. For $F(\omega_+)$ given above a glass solution exists in leading order for $\epsilon(\mu) <$ constant $(\omega_D\tau)^{1/3}(\Gamma\tau/(\pi N_F^2))^{4/3}/(D_0\tau)^2$. For $\mu > \mu_g$ the glass transition temperature first increases with increasing disorder. At the MIT (defined by $\sigma(T = 0) = 0$) the transition temperature has to be zero again. Our methods for calculating J become invalid at the MIT of the reference system. Therefore we can only calculate the increasing part of the transition temperature in figure 1. The dashed line sketches the expected continuation of the transition curve. The glass transition temperature T_g at the MIT of the reference system is in leading order (if $T \ll \omega_D$)

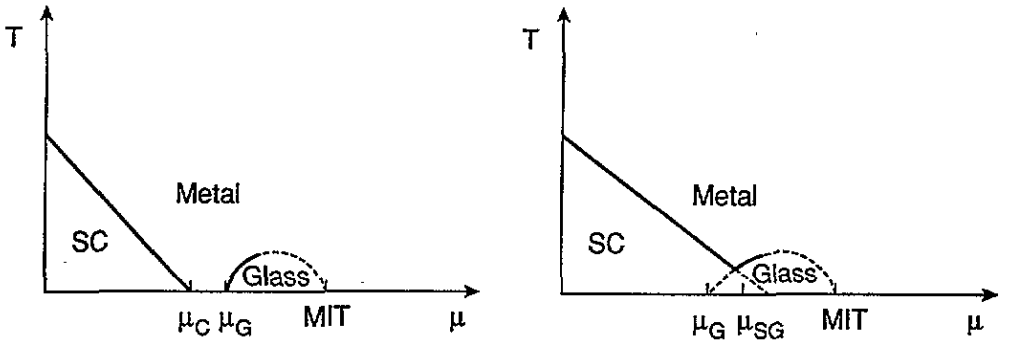

Figure 1. Schematic phase diagrams. Temperature T and disorder μ are given in arbitrary units. Further explanation is given in the text.

$T_g/\omega_D = \frac{1}{2}\exp(-\pi^2 N_F^2 D_0^3 \tau/(\Gamma/N_F)^2)$. The strong dependence on the scale D_0 in the argument of the exponential makes a reliable quantitative estimate impossible. If we take $D_0\tau \approx 10/k_F$, the ratio of Debye energy to Fermi energy $\omega_D/\epsilon_F = 0.01$ and $\Gamma/N_F = 0.4$ and if we assume an experimentally more appropriate form for the conductivity $\sigma(\omega, \epsilon = 0) = \omega^{1/2}$ [28] the resulting transition temperature at $\epsilon = 0$ is in the millikelvin regime. Under the same assumptions but varying $D_0\tau$ between $10/k_F$ and $1/k_F$, $\epsilon(\mu_g)$ changes from 0.0005 to 0.3. Due to the approximation involved, these numbers are only estimates.

In the lower part of figure 1 the scenario for $\mu_g < \mu_c$ is shown. The macroscopic superconductor crosses over to the glass phase at non-zero temperatures for $\mu = \mu_{sg}$. μ_{sg} is implicitly given by $J_0(T_g(\mu_{sg}), \delta\Gamma(\mu_{sg})) = J(T_g(\mu_{sg}), \epsilon(\mu_{sg}))$ at the glass transition temperature $T_g(\mu_{sg})$. The theory does not allow for a superconductor to glass transition for $\mu_g < \mu < \mu_{sg}$, since $J_0(T)$ remains larger than $J(T)$ for decreasing temperature at constant μ. The macroscopic superconductor is unstable for $\mu > \mu_{sg}$, since disorder fluctuations become more effective with increasing disorder.

The aim of this paper was to discuss the possibility of a new glass phase in superconductors and to present the two possible phase diagrams. To actually discuss the critical behaviour of the proposed phase transition, the effects of thermal and quantum fluctuations need to be taken into account. If the fluctuations in the amplitude of $\delta C(x, x')$ were irrelevant the proposed glass phase would be a continuum version of a gauge glass (randomly frustrated X–Y spin glass). [18] indicates that the gauge glass is a thermodynamically stable phase in 3D (see also [29]). Our calculations suggest that this glass phase is realized in disordered superconductors due to the fluctuations in the random phase (gauge) of δC close to the MIT for low enough temperature. Experimentally we expect the usual scaling at the gauge glass transition. Since the gauge glass is in a different universality class than BCS superconductors (X–Y universality class) [19] the critical exponents are different. Consequently it should be possible to clearly distinguish between conventional gapless superconductivity and the glass phase proposed here.

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